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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
FOURTH SEMESTER B.TECH DEGREE EXAMINATION, DECEMBER 2018

Course Code: MA202

Course Name: PROBABILITY DISTRIBUTIONS, TRANSFORMS AND NUMERICAL METHODS

Max. Marks: 100

Duration: 3 Hours

Normal distribution table is allowed in the examination hall.

PART A (MODULES I AND II)

Answer two full questions.

- 1 a) Suppose that the probabilities are 0.4, 0.3, 0.2, and 0.1 that there will be 0, 1, 2, or 3 power failures in a certain city during the month of July. Find the mean and variance of this probability distribution. (7)
- b) During one stage in the manufacture of integrated circuit chips, a coating must be applied. If 70% of chips receive a thick enough coating. Use Binomial distribution to find the probabilities that, among 15 chips
- (i) at least 12 will have thick enough coating;
 - (ii) at most 6 will have thick enough coating;
 - (iii) exactly 10 will have thick enough coating.
- 2 a) If the distribution function of a random variable is given by (7)
- $$F(x) = \begin{cases} 1 - \frac{1}{x^2} & \text{for } x > 1 \\ 0 & \text{for } x \leq 1 \end{cases}$$
- find the probabilities that this random variable will take on a value
- (i) less than 3;
 - (ii) between 4 and 5.
- b) In a given city, 6% of all drivers get at least one parking ticket per year. Use the Poisson approximation to the binomial distribution to determine the probabilities that among 80 drivers (randomly chosen in the city):
- (i) 4 will get at least one parking ticket in any given year;
 - (ii) at least 3 will get at least one parking ticket in any given year;
 - (iii) anywhere from 3 to 6, inclusive, will get at least one parking ticket in any given year.
- 3 a) Derive mean and variance of uniform distribution. (7)
- b) The time required to assemble a piece of machinery is a random variable having approximately a normal distribution with mean 12.9 minutes and standard deviation 2.0 minutes. What are the probabilities that the assembly of a piece of machinery of this kind will take
- (i) at least 11.5 minutes;
 - (ii) anywhere from 11.0 to 14.8 minutes?

PART B (MODULES III AND IV)

Answer two full questions.

- 4 a) Using Fourier cosine integral, show that $\int_0^{\infty} \frac{\cos xw}{1+w^2} dw = \frac{\pi}{2} e^{-x}$ if $x > 0$. (7)
- b) Find the Fourier sine transform of $f(x) = \begin{cases} \sin x & \text{if } 0 < x < \pi \\ 0 & \text{if } x > \pi \end{cases}$. (8)

- 5 a) Find the Fourier transform of $f(x) = \begin{cases} e^{kx} & \text{if } x < 0 \\ 0 & \text{if } x > 0 \end{cases}$, $k > 0$. (7)
- b) Find the inverse Laplace transform of $\frac{5}{(s^2 + 1)(s^2 + 25)}$ using Convolution Theorem. (8)
- 6 a) Find the Laplace transforms of (i) $t e^{kt}$ (ii) $\cos(\omega t + \theta)$ (7)
- b) Solve the initial value problem $y'' - y' - 6y = 0$, $y(0) = 6$, $y'(0) = 13$ by using Laplace transforms. (8)

PART C (MODULES V AND VI)

Answer two full questions.

- 7 a) Find the positive solution of $2\sin x = x$ by using Newton-Raphson method, the solution is near to 2. (7)
- b) Calculate the Lagrange polynomial $p(x)$ for the 4-D values of the function $f(x)$, $f(1.00) = 1.0000$, $f(1.02) = 0.9888$, $f(1.04) = 0.9784$, and from it find the approximate value of $f(x)$ at $x = 1.005$. (7)
- c) Compute $f(1.5)$ from $f(1) = -1$, $f(2) = -1$, $f(3) = 1$, $f(4) = 5$ by using Newton's forward interpolation formula. (6)
- 8 a) Solve $6x_1 + 2x_2 + 8x_3 = 26$, $3x_1 + 5x_2 + 2x_3 = 8$, $8x_2 + 2x_3 = -7$ by Gauss Elimination method. (7)
- b) Find the value of $(13)^{1/3}$ using Newton Raphson method. (7)
- c) Evaluate $\int_0^1 e^{-x^2} dx$ by Trapezoidal rule taking 10 subintervals. (6)
- 9 a) Use Euler's method with $h = 0.1$, compute the value of $y(0.5)$ for the equation $y' = (y + x)^2$, $y(0) = 0$. (7)
- b) Use Runge-Kutta method with $h = 0.1$, compute the value of $y(0.1)$ for the equation $y' = xy^2$, $y(0) = 1$. (7)
- c) Evaluate $\int_0^1 \frac{dx}{\cos^2 x}$ by Simpson's rule taking 10 subintervals and compare it with the exact solution. (6)
