

Reg No: \_\_\_\_\_ Name: \_\_\_\_\_

**APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY**  
**THIRD SEMESTER B.TECH DEGREE EXAMINATION(S), MAY 2019**

**Course Code: CS201**

**Course Name: DISCRETE COMPUTATIONAL STRUCTURES**

Max. Marks: 100

Duration: 3 Hours

**PART A**

*Answer all questions, each carries 3 marks.*

Marks

- |   |   |     |
|---|---|-----|
| 1 | Show that set of all integers are countable.  | (3) |
| 2 | What is the minimum number of students required in a class to be sure that at least six will receive same grade, if there are five possible grades. | (3) |
| 3 | With suitable example define GLB and LUB of a partially ordered set.  | (3) |
| 4 | Show that $(E,+)$ E is set of all even non negative integers is subsemigroup of $(N,+)$ N is set of all natural numbers.                            | (3) |

**PART B**

*Answer any two full questions, each carries 9 marks.*

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|---|--|-----|
| 5 | a) Solve the recurrence relation $a_n = 4a_{n-1} - 4a_{n-2} + (n+1)2^n$  | (5) |
|   | b) Let $R = \{(1,2), (3,4), (2,2)\}$ $S = \{(4,2), (2,5), (3,1), (1,3)\}$ Find $RoS, SoR, Ro(SoR)$ and $RoR$   | (4) |
| 6 | a) Let R be the set of real numbers and S is the relation on R defined by $(x,y) \in S$ if $(x-y)$ is divisible by 7. Prove that S is an equivalence relation. Find equivalent class of S. | (5) |
|   | b) How many permutations are there for the eight letters a,e,f,g,i,t,w,x. How many start with letter 't' and how many start with letter 't' and end with letter 'c'.                       | (4) |
| 7 | a) Check whether the algebraic structure $\langle \mathbb{Z}_5, +_5 \rangle$ defined over the set of positive integers is a semigroup or not?  | (5) |
|   | b) Show that $A \times (B \cap C) = (A \times B) \cap (A \times C)$  | (4) |

**PART C**

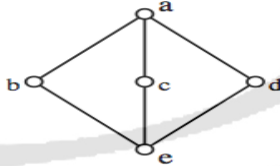
*Answer all questions, each carries 3 marks.*

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|----|---|-----|
| 8  | Prove that inverse element of a group is unique     | (3) |
| 9  | Define algebraic system with two binary operations. | (3) |
| 10 | Prove that every chain is a distributive lattice    | (3) |
| 11 | Define a Boolean algebra. Give an example           | (3) |

**PART D**

*Answer any two full questions, each carries 9 marks.*

- 12 a) State and prove Lagrange's Theorem (5)  
 b) For any Boolean algebra B, prove that  $a+b = a+c$  and  $ab = ac \Rightarrow b = c$  for all  $a, b, c \in B$  (4)
- 13 a) Check whether the following is distributive lattice or not (4)



- b) If  $(a+b)^2 = a^2 + 2ab + b^2 \forall a, b, c \in R$ , Prove that R is a commutative Ring and conversely (5)
- 14 a) Prove that the set  $G = \{0, 1, 2, 3, 4, 5\}$  is a abelian group under addition modulo 6. (5)  
 b) Define lattice homomorphism. (4)

**PART E**

*Answer any four full questions, each carries 10 marks.*

- 15 a) Show that  $n^3 + 2n$  is divisible by 3 by mathematical induction (5)  
 b) Show that the following premises are inconsistent (5)
1. If Jack misses many classes through illness, then he fails high school
  2. If Jack fails high school then he is uneducated
  3. If Jack reads a lot of books, then he is not educated
  4. Jack misses many class through illness and reads a lot of books
- 16 a) Construct truth table for the following formula  $(P \vee Q) \vee (\sim P \vee Q) \vee (P \wedge \sim Q)$  (5)  
 b) Show that  $R \rightarrow S$  can be derived from the premises  $P \rightarrow (Q \rightarrow S)$ ,  $\sim R \vee P$  and  $Q$  (5)
- 17 a) Show that  $(\forall x)(P(x) \vee Q(x)) \Rightarrow (\forall x)P(x) \vee (\exists x)Q(x)$  (5)  
 b) Explain proof by contradiction with an example (5)
- 18 a) Without using truth table prove that  $(P \vee Q) \wedge \sim(\sim P \wedge (\sim Q \vee \sim R)) \vee (\sim P \wedge \sim Q) \vee (\sim P \wedge \sim R)$  is a tautology (5)  
 b) Show that the conclusion C follows from the premises H1, H2  
 H1:  $\sim Q$  H2:  $P \rightarrow Q$  C:  $\sim P$  (5)
- 19 a) Symbolize the expression "All the world loves a lover" (5)  
 b) Prove that  $5+10+15+\dots+5n = 5n(n+1)/2$  using mathematical induction (5)
- 20 a) Determine the validity of the following arguments (5)  
 Every living thing is a plant or animal  
 John's gold fish is alive and it is not a plant  
 All animals have hearts  
 Therefore, John's gold fish has heart.
- b) Show that  $R \wedge (P \vee Q)$  is a valid conclusion from the premises  $P \vee Q, Q \rightarrow R, P \rightarrow M$ , and  $\sim M$  (5)

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