Reg No.:_____ Name:_____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

V SEMESTER B.TECH DEGREE EXAMINATION(S), MAY 2019

Course Code: CS309

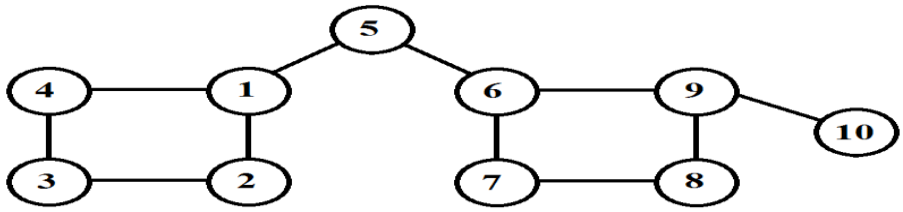
Course Name: GRAPH THEORY AND COMBINATORICS

Max. Marks: 100 Duration: 3 Hours

PART A

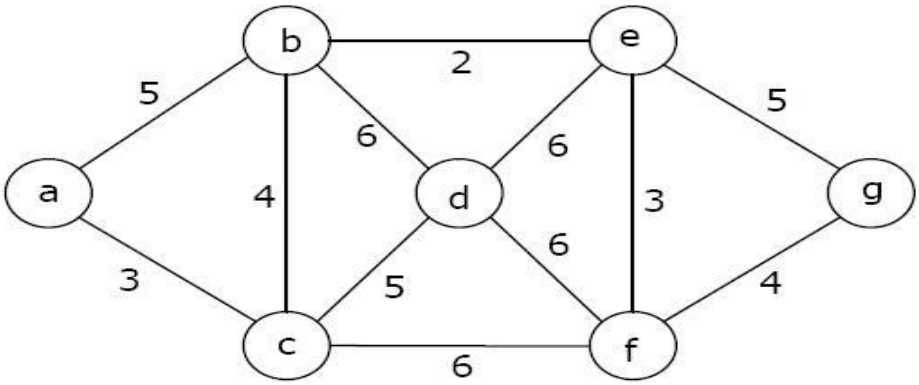
Answer all questions, each carries3 marks. Marks

1 Print a Walk, trail, path and cycle on the graph below. 3



2 Define pendant vertex, isolated vertex and null graph with an example each. 3

3 State travelling salesman problem. Print a travelling salesman's tour on the graph below. 3

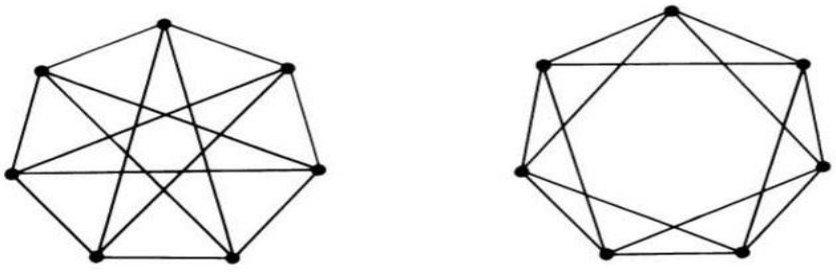


4 Prove Dirac's theorem for Hamiltonicity. 3

PART B

Answer any two full questions, each carries9 marks.

5 a) Define isomorphism of graphs. Show that the graphs (a) and (b) are isomorphic. 4



(a) (b)

b) Define subgraph. Give two subgraphs of the above graph.(Fig. a) 2

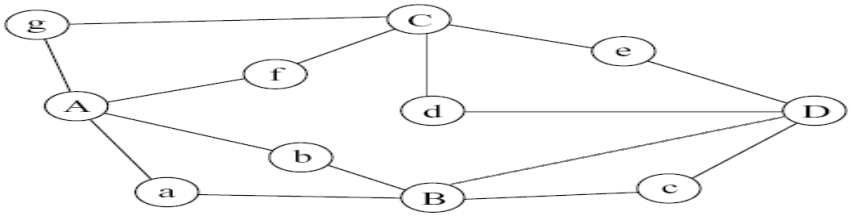
c) Consider a complete graph G with 11 vertices. 3

1. Find the maximum number of edges possible in G
2. Find the number of edge-disjoint Hamiltonian circuits in G

6 a) Draw a simple disconnected graph with 10 vertices, 4 components and maximum number of edges. 3

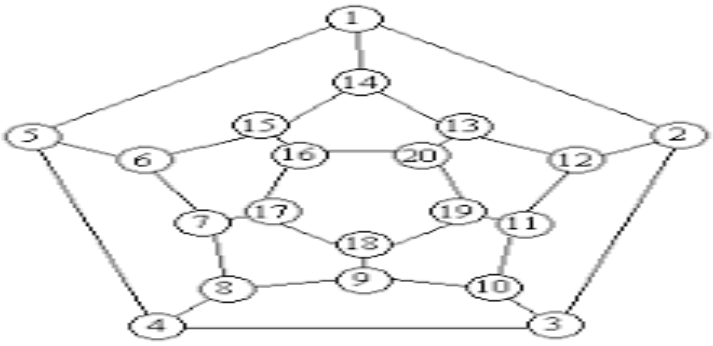
b) Explain any two applications of graphs. 2

c) Check whether the given graph is an Euler graph and if yes, give the Euler line. Justify your answer. 4



7 a) Prove or disprove: If every vertex of a simple graph G has degree 2, then G is a cycle. 3

b) Give Hamiltonian circuit of the following graph. 3



c) In a graph G let p1 and p2 be two different paths between two given vertices. Prove that ringsum of p1 and p2 is a circuit or a set of circuits. 3

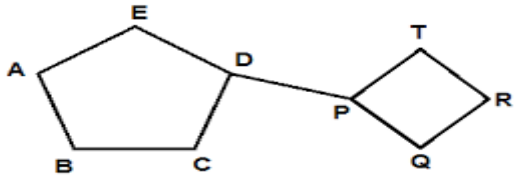
PART C

Answer all questions, each carries3 marks.

8 Prove that in a graph G, if there is exactly one path between every pair of vertices, then G is a tree. 3

9 Given a spanning treeof a graph, how will you find out all spanning trees ? 3

10 List all cut sets of the following graph. 3



11 Prove that every circuit has an even number of edges in common with any cut set. 3

PART D

Answer any two full questions, each carries9 marks.

12 a) Define a tree. Give any 4 properties of trees. 3

b) Prove that a graph is a tree if and only if it is loop-less and has exactly one spanning tree. 3

c) Prove that every circuit has an even number of edges in common with any cut set. 3

13 a) Prove that every tree has either one or two centers. 3

b) Write short notes on geometric dual and combinatorial dual. 6

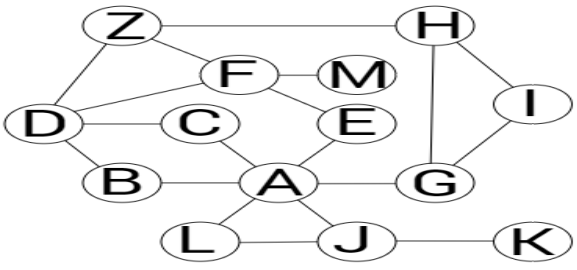
14 a) Draw a connected graph G and find two spanning trees T1 nad T2 of G such that the distance (T1, T2)= 3. Find the branch set, chord set, rank and nullity of T1. 4

b) Construct a graph G with the following properties: Edge connectivity=4, Vertex connectivity =3 and degree of every vertex of G is greater than or equal to 5. 5

PART E

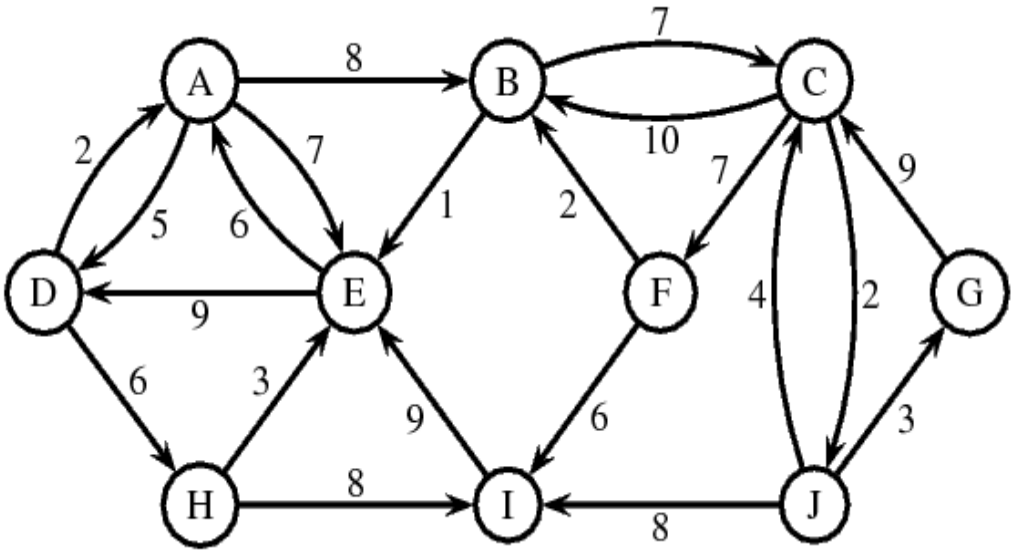
Answer any four full questions, each carries10 marks.

15 a) Give incidence matrix of the following graph. 3



b) Prove that two graphs G1 and G2 are isomorphic if and only if their incidence matrices A(G1) and A(G2) differ only by permutations of rows and columns. 2

c) Give Dijkstra's algorithm to find shortest path between a vertex pair. Use it to find shortest path between A and G. 5



16 a) Prove that if B is a circuit matrix of a connected graph G with n vertices and e edges, then rank of B is e-n+1. 3

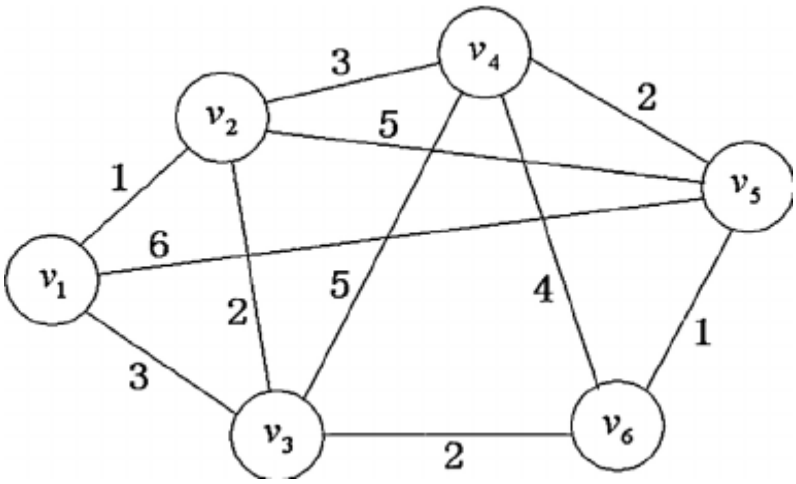
b) How will you get fundamental circuit matrix from a circuit matrix. Derive the rank of a fundamental circuit matrix. 2

c) Explain successor listing and incidence matrix methods used in computer representation of a graph? 5

17 a) Prove that the rank of cut set matrix C(G) is equal to rank of the incidence matrix A(G), which equals the rank of the graph G. 3

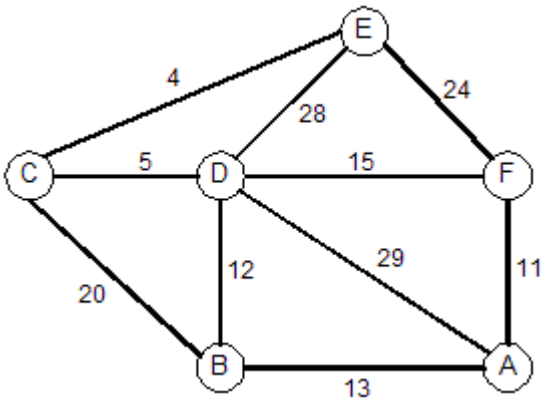
b) Define path matrix. What is the disadvantage of path matrix compared to other matrices. 2

c) Find a minimum spanning tree of the following graph. Also give its rank and nullity. 5

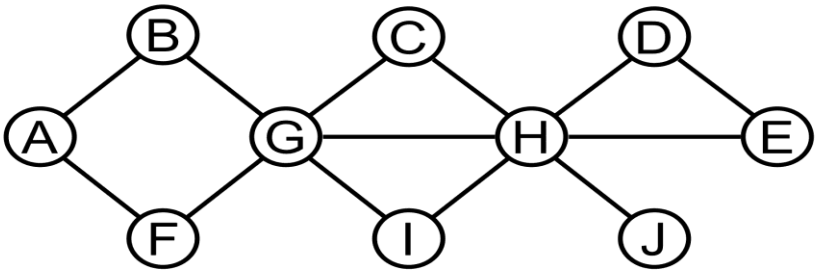


18 a) If A(G) is an incidence matrix of a connected graph G with n vertices, then the rank of A(G) is n-1. 5

b) How is Kruskal's algorithm used to find minimum cost spanning tree of a graph. Find a minimum spanning tree in the graph below. 5



19 a) Write cut set matrix of the following graph. Give its rank. 5



b) Give an algorithm to check whether a graph is connected or not. How it can be implemented with an adjacency matrix. 5

20 a) Give any five properties of circuit matrix. 5

b) How are edge listing and linear arrays used in computer representation of a graph? 5