

PART B*Answer any two full questions, each carries 15 marks*

- 4 a) The life time of a certain type of electric bulbs may be considered to follow exponential distribution with mean 50 hrs. Use central limit theorem to find the approximate probability that 100 of these electric bulbs will provide a total of more than 6000 hrs of burning time. (7)

- b) The joint density function of two continuous random variables X, Y is given by (8)

$$f(x, y) = \begin{cases} K(1 - x - y), & 0 < x < \frac{1}{2}; 0 < y < \frac{1}{2} \\ 0, & \text{otherwise} \end{cases}$$

Find (i) the value of K (ii) $P\left(X < \frac{1}{4}, Y > \frac{1}{4}\right)$ (iii) the marginal distributions of X, Y (iv) check whether X, Y are independent.

- 5 a) Let $X(t) = A\cos\omega t - B\sin\omega t$, where A and B are independent random variables following $N(0, \sigma^2)$. Then show that $\{X(t)\}$ is WSS. (7)

- b) Find the power spectral density function of the WSS process whose autocorrelation function is $e^{-\alpha\tau^2}$. (8)

- 6 a) The joint probability distribution of X and Y is given by $f(x, y) = \frac{2x+3y}{54}$ for $x = 1, 2; y = 1, 2, 3$. Find (i) the marginal distributions of X and Y (ii) The conditional distribution of X for $Y = y$. (7)

- b) The power spectral density of a WSS process is $\frac{\omega^2+9}{\omega^4+5\omega^2+4}$. Find the autocorrelation function and power of the process. (8)

PART C*Answer any two full questions, each carries 20 marks*

- 7 a) The tpm of a Markov chain with 4 states 0, 1, 2, 3 is given by

$$P = \begin{bmatrix} 0.2 & 0.8 & 0 & 0 \\ 0 & 0.2 & 0.8 & 0 \\ 0 & 0 & 0.2 & 0.8 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (10)$$

with initial distribution $P\{X_0 = i\} = \frac{1}{3}, i = 0, 1, 2, 3$.

Find (i) $P\{X_1 = 2 / X_0 = 1\}$ (ii) $P\{X_2 = 3 / X_0 = 1\}$

(iii) $P\{X_2 = 3, X_1 = 2, X_0 = 2\}$ (iv) $P\{X_2 = 3\}$

- b) The tpm of a Markov Chain is $P = \begin{bmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$. Find the steady state distribution of (5)

the chain.

- c) A radioactive source emits particles at the rate of 6 per minutes in accordance with (5)
Poisson process. Each particle emitted has a probability of $\frac{1}{3}$ being recorded.
Find the probability that atleast 5 particles are recorded in 5 minutes.

- 8 a) Evaluate $\int_4^{5.2} \log_e(x) dx$ using Simpson's $1/3^{\text{rd}}$ rule . (Take $h = 0.2$) (7)

- b) Use Newton's forward interpolation formula to evaluate $y(23)$ from the following (7)
data:

x	20	25	30	35	40	45
y	34.3	32.1	29.3	25.6	22.7	21.9

- c) Use Runge-Kutta method of order 4 to find $y(0.2)$ for the differential equation: (6)
 $y' = 3x + 0.5y, y(0) = 1$. (Take $h = 0.2$)

- 9 a) A house wife buys 3 types of cereals A, B, and C. She never buys the same cereals (10)
in successive weeks. If she buys cereal A, next week she buys B. However, if she
buys B or C, next week she is 3 times as likely to buy A as the other cereals. In the
first week of May she buys cereal C. Then what is the probability that (i) in the
second week she buys cereal A (ii) In the third week she buys cereal C (iii) In
the long run, how often she buys cereal B.

- b) Use Lagrange's interpolation formula to find $y(2)$ from the following table: (5)

x	1	3	4
y	1	27	64

- c) Evaluate cube root of 41 correct to four decimal places using Newton- Raphson (5)
method correct to 4 decimal places.