PART B

Answer any two full questions, each carries 15 marks.

4 a) Derive the relation between Laplace transform and Continuous Time Fourier transform. (3)

b) Evaluate the Fourier Transform of $x(t) = sgn(t)$. Plot magnitude and phase response. (3)

c) An LTI system is characterized with the transfer function $H(s) = \frac{s+5}{s^2+3s+2}$. Find the response of the system to the input $x(t) = \cos 2t\, u(t)$. (5)

d) State Sampling theorem. Compute the Nyquist rate of the signal $x(t)$. (4)

$$x(t) = \cos\left(\frac{\pi t}{2}\right) - \sin\left(\frac{\pi t}{8}\right) + \cos\left(\frac{\pi t}{4} + \frac{\pi}{3}\right)$$

5 a) Determine the Fourier Series Representation for $x(t)= 2\sin(2\pi t-3) + \sin(6\pi t)$. (6)

b) Show that the spectrum of the sampled signal is the infinite sum of shifted replicas of the spectrum of original signal. (6)

c) Evaluate the Fourier Transform of $x(t) = \frac{d(te^{-2t}\sin(t)u(t))}{dt}$. (3)

6 a) A causal LTI system has an impulse response $h(t) = e^{-4t}\, u(t)$. Using Fourier transform find, (7)

(i) Frequency response of the system.

(ii) Output of the system for an input $x(t) = 3e^{-t}\, u(t)$.

b) State and prove the following properties of Laplace Transform (4)

(i) Time domain differentiation

(ii) Final value theorem

c) Find the Inverse Fourier transform of the following signals (4)

(i) $\frac{1}{j\Omega(j\Omega+1)} + 2\pi\delta(\Omega)$

(ii) $2\pi\delta(\Omega) + \pi\delta(\Omega - 4\pi) + \pi\delta(\Omega + 4\pi)$

PART C

Answer any two full questions, each carries20 marks.

7 a) Find the Z - transform of $x(n) = 2(3)^n u(-n)$ (5)

b) Compute the DTFT of the signal *x(n)*. (4)

$$x(n) = \begin{cases} 10 & ; |n| \le N \\ 0 & ; |n| > N \end{cases}$$

c) Prove that, for a BIBO stable discrete time LTI system the ROC of system function includes unit circle. (3)

d) An LTI system is described by the following input-output relation (8)

$y(n) - \frac{9}{4}y(n-1) + \frac{1}{2}y(n-2) = x(n) - 3x(n-1).$

Determine the impulse response of the system with specified ROCs of H(z) for the conditions:

(i) System is stable (ii) System is causal

8 a) Find the discrete time Fourier series coefficients of the signal $x(n) = 5 + sin\left(\frac{n\pi}{2}\right) + cos\left(\frac{n\pi}{4}\right)$. Plot the magnitude and phase spectrum. (6)

b) Find all possible time domain signals for the Z- transform $X(z) = \frac{1}{1-\frac{1}{6}z^{-1}-\frac{1}{6}z^{-2}}$. (6)

c) A stable and causal LTI system produces an output $y(n) = n\left(\frac{4}{5}\right)^n u(n)$, for the excitation $x(n) = \left(\frac{4}{5}\right)^n u(n)$. Using Discrete Time Fourier transform, (8)

(i) Determine the Frequency response of the system.

(ii) Derive the difference equation relating the input and output.

9 a) Using Z- transform, determine the output of an LTI system with impulse response $h(n) = \{1, 2, -1,0,3\}$ for the input $x(n) = \{1, 2, -1\}$. (3)

b) Determine the Discrete Time Fourier transform of $x(n) = \left(\frac{1}{2}\right)^n \sin\left(\frac{n\pi}{4}\right) u(n)$. (4)

c) Compute the Z-transform and ROC of the signal $x(n) = \left(\frac{1}{2}\right)^n u(-n) - 2^n u(-n-1)$. Plot the pole-zero pattern. (8)

d) Mathematically explain how DTFT is related with Z- transform. (5)