PART B

Answer any two questions

4 a) Obtain the distribution function of a continuous two dimensional random variable (X,Y) with the joint pdf given by 7

$$f(x,y) = \begin{cases} e^{-x-y}, & 0 < x < \infty\ , 0 < y < \infty \\ 0, & \text{elsewhere} \end{cases}$$

b) If the joint pdf of (X,Y) is given by 8

$$f(x,y) = \begin{cases} \tfrac{1}{2}, & x > 0, y > 0, x + y < 2 \\ 0, & \text{otherwise} \end{cases}$$

Find $P\{X \le 1, Y \le 1\}$, $P\{ X + Y < 1 \}$ and $P \{ X > 2Y \}$

5 a) The joint pdf of (X,Y) is 7

$$f(x,y) = \begin{cases} 8xy, & 0 < y < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

i) Check whether X and Y are independent

ii) Find $P(X + Y < 1)$

b) Prove that the power spectral density and autocorrelation function of a real WSS process form a Fourier cosine transform pair 8

6 a) If $X(t) = P + Qt$ is the random process where P and Q are independent random variables with $E(P)= p$, $E(Q)= q$, $Var(P)= \sigma_1^2$, $Var (Q) = \sigma_2^2$ then find the mean, autocorrelation and autocovariance of the process 7

b) Consider the random process $X(t)= A \cos(\omega t) + B \sin (\omega t)$ where A and B are independent random variables with mean 0 and equal variance. Show that X(t) is a WSS. 8

PART C

Answer any two questions

7 a) Cell phone calls processed by a certain wireless base station arrive according to a Poisson process with an average of 12 per minute. i) What is the probability that more than two calls arrive in an interval of length 20 seconds ii) What is the probability that more than 2 calls arrive in each of two consecutive intervals of length 30 seconds 7

b) Show that the time between any two consecutive occurrences of a Poisson process is a random variable following an exponential distribution 7

c) Find the mean, variance, autocorrelation and autocovariance of a Poisson process 6

8 a) Find the positive solution of the equation 2sinx = x using Newton-Raphson method 6

b) Using Newton's forward difference interpolation formula evaluate f(2.05) from the following table 6

X	2.0	2.1	2.2	2.3	2.4
f(x)	1.414214	1.449138	1.483240	1.516575	1.549193

c) Find an approximate value of $\log_e 5$ by evaluating $\int_0^5 \frac{dx}{4x+5}$ using Simpson's 1/3rd rule with h=0.5 8

9 a) The transition probability matrix of a Markov chain $\{X_n\}$, n =1,2,3,….. having three states 1,2 and 3 is $P = \begin{bmatrix} 0.1 & 0.5 & 04 \\ 0.3 & 0.4 & 0.3 \\ 0.6 & 0.2 & 0.2 \end{bmatrix}$ 10

And the initial distribution is

P(0)= [0.7 0.2 0.1]. Find $P(X_2=3)$

b) Evaluate $\int_0^1 \frac{dx}{1+x}$ using trapezoidal rule 5

c) Using Runge-Kutta method of order four, find y(0.2) given that $\frac{dy}{dx} = y - x$, y(0)=2 by taking h=0.1 5