

10103

Reg. No: \_\_\_\_\_

Name: \_\_\_\_\_

**APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY**  
**FIRST SEMESTER B.TECH DEGREE EXAMINATION, JUNE 2016**

**Course Code: MAJ01**

**Course Name: CALCULUS**

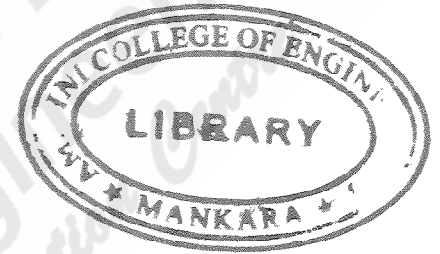
Max. Marks: 100

Duration: 3 Hours

**PART A**

*Answer ALL questions. Each question carries 3 marks*

1. Evaluate  $\int_0^1 \sinh^2(2x) dx$
2. Check whether the series  $\sum_{k=1}^{\infty} \frac{1}{2k-1}$  converges or not.
3. Identify the quadric surface  $6x^2 + 3y^2 + 4z^2 = 24$
4. Convert  $(2\sqrt{3}, \pi/3, 6)$  from cylindrical to spherical co-ordinates.
5. Find the rate of change of  $f(x,y)=xe^{-y} + 5y$  with respect to  $x$  at the point  $(4,0)$  with  $y$  held fixed.
6. If  $f(x,y) = x^2y^3 + x^4y$ , Find  $f_{xy}$
7. Evaluate  $\int_1^9 \left( \left(\frac{t}{2}\right) i + \left(t - \frac{1}{2}\right) j \right) dt$
8. Find  $\frac{d\vec{u}}{dt}$  if  $\vec{u}(t) = (3t i + 5 t^2 j + 6 k) \cdot (t^2 i + 2t j + t k)$
9. Sketch the region of integration in  $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2) dy dx$
10. Evaluate  $\int_0^1 \int_0^1 \int_0^1 e^{(x+y+z)} dx dy dz$



PART B

Answer any 2 complete questions each having 7 marks

11. A ball is dropped from a height of  $h$  feet and on each bounce rises 75% of the distance it has fallen previously. If it travels a distance of 21 feet what is  $h$ ?

12. Use Ratio Test for absolute convergence to find whether the series

$$\sum_{k=1}^{\infty} \frac{(-1)^{k+1} 2^k}{k!} \text{ converges.}$$

13. Find the Maclaurin's Series for  $\frac{1}{1-x}$

Answer any 2 complete questions each having 7 marks

14. For the surface  $4x^2 + 9y^2 + 18z^2 = 72$

a. Find the equation of the elliptical trace in the plane

b.  $z = \sqrt{2}$

c. Find the length of the major and minor axes of the ellipse.

15. Find  $\lim_{(x,y) \rightarrow (0,0)} (x^2 + y^2) \ln(x^2 + y^2)$

16. Let  $f(x, y) = \frac{x^2 - y^2}{x^2 + y^2}$ . Determine the limit of  $f(x, y)$  as

$(x, y)$  approaches  $(0, 0)$  along the curve  $C$ , where  $C$  is

(a)  $x = 0$  (b)  $\dot{y} = 0$  (c)  $y = x$

(d)  $y = x^2$  (e)  $x = y^2$

Answer any 2 complete questions each having 7 marks

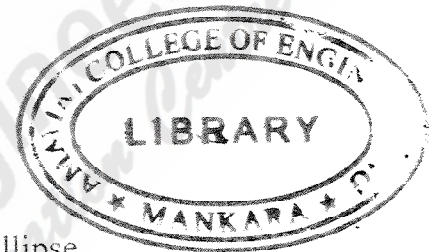
17. Use chain rule to find  $\frac{dw}{ds}$  at  $s = 1/4$  if

$$w = r^2 - r \tan \theta; \quad r = \sqrt{s}; \quad \theta = \pi s$$

18. Locate all relative extrema and saddle points of  $f(x,y) = x^2 + xy - 2y - 3x + 1$

19. The volume  $V$  of a right circular cone of radius  $r$  and height  $h$  is given by  $V =$

$$\frac{1}{3} \pi r^2 h. \text{ Suppose that the height decreases from 20 to 19.95 units and the radius}$$



increases from 4 to 4.05 units. Compare the change in volume of the cone with an approximation of this change using a total differential.

*Answer any 2 complete questions each having 7 marks*

20. The temperature in degree Celsius at a point in the  $(x, y)$  plane is

$$T(x, y) = \frac{xy}{1 + x^2 + y^2}$$

Find the rate of change of temperature at  $(1, 1)$  in the direction of  $(2\hat{i} - \hat{j})$ .

21. Find the scalar tangential and normal components of acceleration at time  $t$  of a

particle with position vector at time  $t$  is  $r(t) = t\hat{i} + t^2\hat{j} + t^3\hat{k}$

22. Find the equation of the tangent plane and parametric equation for the normal line to the surface  $x^2 + y^2 + z^2 = 25$  at  $P(3, 0, 4)$

*Answer any 2 complete questions each having 7 marks*

23. Evaluate  $\iint_R \sin \theta \, dA$  where  $R$  is the region in the first quadrant that is outside the

circle  $r = 2$  and inside the cardioid  $r = 2(1 + \cos \theta)$ .

24. Find the Jacobian  $\frac{\partial(x, y, z)}{\partial(u, v, w)}$  where  $x = 4u + v$ ,  $y = u - 2w$ ,

$$z = v + w.$$

25. By changing the order of integration evaluate  $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} \, dy \, dx$

