10252

Reg. No. Name:

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY SECOND SEMESTER B.TECH DEGREE SPECIAL EXAMINATION, AUGUST 2016 Course Code: MA-102

Course Name: DIFFERENTIAL EQUATIONS

Max. Marks: 100 Duration: 3 hrs

PART A

Answer all questions Each carries 3 marks

- 1. Find ordinary differential equation for the basis $e^{-x\sqrt{2}}$, $xe^{-x\sqrt{2}}$
- 2. Reduce y'' = y' to 1st order differential equation and solve.
- 3. Find the particular solution to $(D^4 m^4)$ $y = \sin mx$
- 4. Use variation of parameters to solve $y'' + y = \sec x$
- 5. Find the Fourier coefficient a_n for the function f(x) = 1 + |x| defined in -3 < x < 3
- 6. Develop the Fourier Sine series of f(x) = x in $0 < x < \pi$
- 7. Obtain the partial differential equation by eliminating arbitrary function from $x^2 + y^2 + z^2 = f(xy)$
- 8. Solve $y^2zp + x^2zq = xy^2$
- 9. Solve $u_x + u_y = 0$ using method of separation of variables
- 10. A finite string of length L is fixed at both ends and is released from rest with a displacement f(x). What are the initial and boundary conditions involved in this problem?
- 11. Write all the possible solutions of one-dimensional heat transfer equation
- 12. Find the steady state temperature distribution in a rod of length 30cm having the ends at 20° C and 80° C respectively.

PART B Answer one full question from each module Module -I

13. (a) Verify linear independence of $e^{-x}\cos x$ and $e^{-x}\sin x$ using Wronskian and hence solve the initial value problem y'' + 2y' + 2y = 0, y(0) = 0, y'(0) = 15

(b) Find the general solution of the equation
$$x^2y'' + xy' + (x^2 - 0.25)y = 0$$

If $y_1 = \frac{\cos x}{\sqrt{x}}$

OR

14. (a) Find a second order homogeneous linear ODE for which x, $x \log x$ are solutions and solve the IVP with y(1) = 2, y'(1) = 4.



(b) Solve the IVP y'' - 4y' + 9y = 0, y(0) = 0, y'(0) = -8

Module- II

(a) Solve $(D^2 - 2D + 5)y = e^{2x} \sin x$

(b) Solve
$$((x+1)^2 \frac{d^2y}{dx^2} + (x+1) \frac{dy}{dx} = (2x+3)(2x+4)$$

OF

16. (a) Solve
$$x^3 \frac{d^3 y}{dx^3} + 2x^2 \frac{d^2 y}{dx^2} + 2y = 10 \left(x + \frac{1}{x} \right)$$

(b) Solve
$$y'' - 4y' + 5y = \frac{e^{2x}}{\sin x}$$
 by method of variation of parameters

Module - III

- 17. (a) Find the Fourier series representation of $f(x) = x \sin x$ periodic with period 2π , defined in $0 < x < 2\pi$
 - (b) Find the Fourier cosine series of $f(x) = \cos x$, $0 < x < \frac{\pi}{2}$

OR

- 18. (a) Find the Fourier series expansion of $f(x) = e^{-x}$ in -c < x < c
 - (b) Develop the Sine series representation of $f(x) = \begin{cases} x & 0 < x < 2 \\ 4 x & 2 < x < 4 \end{cases}$

Module - IV

19. (a) Solve
$$(y + zx)p - (x + yz)q = x^2 - y^2$$

(b) Find the differential equation of all spheres of fixed radius having their centres in XY -plane.

OF

20. (a)Solve
$$(D^2 - 2DD - 15D^2)z = 12xy$$

(b) Find the particular integral of

$$(D^3 - 7DD^{'2} - 6D^{'3})z = \sin(x + 2y) + e^{3x+y}$$

Module - V

21. A tightly stretched homogeneous string of length 20cm with its fixed ends executes transverse vibrations. Motion starts with zero initial velocity by displacing the string into the form $f(x) = K(x^2 - x^3)$. Find the deflection u(x,t) at any time t.

OR

22. A tightly stretched string of length 'a' with fixed ends is initially in equilibrium position. Find the displacement u(x,t) of the string if it is set vibrating by giving each of its points a velocity $v_0 sin^3 \left(\frac{\pi x}{a}\right)$.

Module - VI

23. Find the temperature distribution in a rod of length 2m whose end power amount maintained at temperature zero and the initial temperature $f(x) = 100(2x - x^2)$

OR

24. The temperatures at the ends of a bar of length l cm with insulated sides are $30^{\circ}C$ and $90^{\circ}C$ respectively until steady state conditions prevail. If the temperature at each end is then suddenly reduced to $0^{\circ}C$ and maintained so, find the temperature distribution at a distance x at time t.