D 27297 (Pages : 2) Name...

Reg. No...

THIRD SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, DECEMBER 2006

EN 04 301 A—ENGINEERING MATHEMATICS

(2004 admissions—Common to all branches except CS and IT)

Time : Three Hours Maximum : 100 Marks

Answer **all** *questions*

1. (a) Define Linear dependence and independence.

   (b) Let $v = \{(x, y, z) \in R^3 : x - 2y + 3z = 0\}$ then show that v is a subspace of R^3.

   (c) Express $f(x) = \begin{cases} 1 & \text{for } |x| \leq 1 \\ 0 & \text{for } |x| > 1 \end{cases}$ as a Fourier integral.

   (d) Find the Fourier sine and cosine transform of $f(x) = e^{-ax}$ $(a > 0)$

   (e) Find P (X < 4) and P (X ≥ 5), from the probability density function of a variate "X" in table :

X	:	0	1	2	3	4	5	6
P (X)	:	K	$3k$	$5k$	$7k$	$9k$	$11k$	$13k$

   (f) If a random variable has a Poisson distribution such that P (1) = P (2) find the mean of the distribution.

   (g) What is the use of goodness of fit test ? (χ^2-test of goodness of fit).

   (h) Give the 95 % confidence interval of the population mean in terms of the mean and SD of a small sample.

(8 × 5 = 40 marks)

2. (a) (i) Obtain an orthonormal basis from the vectors (2, 3, –6) (3, 0, 4) and (6, 3, – 2).

(7 marks)

   (ii) Find a basis and the dimension of the subspace W of R^4 generated by (1, –4, –2, 1), (1, –3, –1, 2) and (3, –8, –2, 7).

(8 marks)

Or

   (b) (i) Explain how will you find an orthonormal basis from a given set of non-zero independent vector's.

(7 marks)

   (ii) Verify that the following is an inner product in R

$$\langle u, v \rangle = x_1 y_1 - x_1 y_2 - x_2 y_1 + 3 x_2 y_2$$

where $u = (x_1, x_2)$ and $v = (y_1, y_2)$.

(8 marks)

Turn over

3. (a) (i) Find the Fourier transform of $f(x) = \begin{cases} 1 & \text{for } |x| < 1 \\ 0 & \text{for } |x| > 1 \end{cases}$. Hence evaluate $\int_0^\infty \frac{\sin x}{x}\, dx$.

(7 marks)

(ii) Show that using Fourier integral representation $\int_0^\infty \frac{\cos wx}{1 + w^2}\, dw = \frac{\pi}{2} e^{-x}\ (x \geq 0)$.

(8 marks)

Or

(b) Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2 & |x| \leq 1 \\ 0 & |x| > 1 \end{cases}$. Hence evaluate $\int_0^\infty \frac{x \cos x - \sin x}{x^3} \cos \frac{x}{2}\, dx$.

(15 marks)

4. (a) (i) A random variable x has the following probability function :—

x	-2	-1	0	1	2	3
P (x) :	0.1	k	0.2	$2k$	0.3	k

Find the value of k, and calculate mean and variance.

(7 marks)

(ii) Fit a normal curve to the following distribution :—

x	2	4	6	8	10
$f(x)$	1	4	6	4	1

(8 marks)

Or

(b) (i) If X and Y are independent binomial random variables having respective parameters (n, p) and (m, p). Prove that the conditional probability mass function of X, given that $X + Y = k$, is that of a hyper geometric random variable.

(7 marks)

(ii) If X and Y are independent random variable following N (8, 2) and $N\left(12, 4\sqrt{3}\right)$ respectively. Find the value of λ such that $P(2X - Y \leq 2\lambda) = P(X + 2Y \geq \lambda)$.

(8 marks)

5. (a) (i) 15.5 % of a random sample of 1600 undergraduates were smokers. Where as 20 % of a random sample of 900 post graduates were smokers in a state. Can we conclude that less number of undergraduates are smokers than the post graduates ?

(7 marks)

(ii) In a random sample of size 500, the mean is found to be 20. In another independent sample of size 400, the mean is 15. Could the samples have been drawn from the same population with S.D. 4 ?

(8 marks)

Or

(b) Fit a binomial distribution for the following data and also test the goodness of fit :—

x :	0	1	2	3	4	5	6	total
f :	5	18	28	12	7	6	4	80

(15 marks)

[4 × 15 = 60 marks]