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Name..................................

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THIRD SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, DECEMBER 2008

EN 04 301—A ENGINEERING MATHEMATICS—III

(2004 Admissions)

[Common to all Branches except CS/IT]

Time : Three Hours

Maximum : 100 Marks

Answer **all** *questions.*

I. (a) Show that the vectors (1, 2, 3), (3, 2, 1) and (5, 5, 5) in $R^{(3)}$ are linearly dependent.

(b) Find the basis and the dimension of the sub-space generated by (2, 3, −1, 2), (−1, 1, 3, 0) and (7, 6, −11, [illegible]) in $R^{(4)}$.

(c) Find the Fourier sine transformation of $e^{-x} + e^{-2x}$.

(d) Using Fourier integral prove that :

$$\int_0^{\infty} \frac{1 - \cos \pi x}{x} \sin xt \, dx = \begin{cases} \frac{\pi}{2} & 0 < t < \pi \\ 0 & t > \pi \end{cases}$$

(e) A random variable X has the probability distribution given below :

X	0	1	2	3	4	5
P(X)	$a/2$	$3a/2$	$2a$	a^2	$2a^2$	$2a^2$

Find the value of 'a' and P (X ≥ 3).

(f) Find the mean and variance of the geometric distribution

$P_x = \frac{2}{3^{x+1}}$, $x = 0, 1, 2, 3, \ldots$ upto ∞.

(g) The mean weight of a random sample of size 100 is 63 gms. If the standard deviation of the weight of the population is 3 gms, find a 95% confidence limits of the mean weight of the population.

(h) The average marks in Mathematics I of a sample of 90 students was 56 and the standard deviation was 5 marks. Test the hypothesis that the average marks of the Mathematics I of the population is 58.

(8 × 5 = 40 marks)

Turn over

2. (a) (i) Prove that the vectors (1, 1, [illegible]) [illegible] 1, 0, 1), (0, 1, 1, 1) and (1, 1, 1, 1) form a basis of vector space $R^{(4)}$. Express (2, 1, 1, [illegible]) as a linear combination of the basis.

(7 marks)

(ii) Find a basis and dimension of the sub-space generated by the vectors (2, 1, 1, − 1), (0, 1, 2, 2) (1, − 1, 1, 0) and (1, [illegible] 1, [illegible]) in $R^{(4)}$.

(8 marks)

Or

(b) (i) Verify whether the following represents an inner product in R^2: $\langle u, v \rangle = 2x_1 y_1$ [illegible] x_2 [illegible] + [illegible] x_2 where $u = (x_1, x_2)$ and $v = (y_1, y_2)$.

(7 marks)

(ii) Find an orthonormal basis for the set of basis (1, 1, 1), (1, − 1, 0) and (0, 1, −1) in $R^{(3)}$.

(8 marks)

3. (a) (i) Find the Fourier transform of [illegible]

(7 marks)

(ii) Find the Fourier cosine transform of [illegible] $x > 0$.

(8 marks)

Or

(b) (i) Using Fourier integral show that $\int_0^\infty \frac{\cos \lambda x}{1+\lambda^2} \, d\lambda = \frac{\pi}{2} e^{-x}, x \geq 0$.

(7 marks)

(ii) Find the Fourier sine transform of [illegible]

(8 marks)

4. (a) (i) Find the value of k, the probability distribution function and P[X < 7] of the density function

$$f(x) = \begin{cases} kx^2 & 0 \le x \le 5 \\ 5k(10 - x) & 5 \le x \le 10 \end{cases}$$

(7 marks)

(ii) In a factory 10% of the rivets manufactured are defective. In a random chosen 10 rivets. What the probability that (i) not more than one rivet is defective ; (ii) atleast 3 rivets are defectives.

(8 marks)

Or

(b) (i) In a normal distribution 10% of the items are below [illegible] and 20% are above 59. Find the mean and variance of the normal distribution.

(7 marks)

(ii) The weights of certain brand shampoo packets are uniformly distributed between 9.4 gms to 10.35 gms. In a random lot of 100 packet how many packets : (i) exceed 10 gms (ii) below 10.2 gms.

(8 marks)

5. (a) (i) A random sample of heights of 1,900 soldiers has a mean of 69 inches and S.D. of [illegible] inches and a random sample of heights of 1600 sailors has a mean of 67.95 inches and S.D. of 2.9 inches. Test whether the heights of soldiers and sailors differ significantly.

(7 marks)

(ii) The content of nitrate in every quintal bag of certain fertilizer is set at 12 kg. In a sample of 10 such bags the content of nitrate are found to be 11, 11, 13, 13, 11, 12, 12, 10, 11, 10. Is there reason to believe that the differce from the set value significant ?

(8 marks)

Or

(b) Fit a Poisson distribution for the following data and test the goodness of fit :—

x	:	0	1	2	3	4	5	6
f	:	278	73	29	6	5	3	1

(15 marks)

[4 × 15 = 60 marks]