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D 26574 (Pages : 2) Name..............................

Reg. No..............................

COMBINED FIRST AND SECOND SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, DECEMBER 2006

EN 04–101—ENGINEERING MATHEMATICS—I

(2004 admissions)

Time : Three Hours Maximum : 100 Marks

Answer **all** *questions.*

Part A

I. (a) Find the radius of curvature at the point (x, y) of the curve $x^{2/3} + y^{2/3} = a^{2/3}$.

(b) Verify Euler's theorem on homogeneous functions for the function $u = \left(x^2 + y^2 + z^2\right)^{-1/3}$.

(c) Discuss the convergence of the series $\sum_{n=1}^{\infty}\left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}\right)$.

(d) Expand $\log(1 + x)$ using Maclaurin's series.

(e) Find the rank of the matrix by reducing it to the normal form $\begin{bmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \\ 9 & 10 & 11 & 12 \end{bmatrix}$.

(f) Find the values of a and b for which the equations $x + y + z = 3$, $x + 2y + 2z = 6$, $x + ay + 3z = b$ have (i) no solution ; (ii) a unique solution.

(g) Obtain the Fourier series expansion for $f(x) = \begin{cases} 1, & 0 < x < l/2 \\ 0, & l/2 < x < l \end{cases}$.

(h) Expand $f(x) = e^{-x}$ as a Fourier series in $(-l, l)$.

($8 \times 5 = 40$ marks)

Part B

II. (a) (i) Find the evolute of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. (8 marks)

(ii) Find the centre of curvature of the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$. (7 marks)

Or

(b) (i) The area of a triangle is calculated from the angles A and C and the side b. If δA is the error in measuring A, show that the relative error in the area is approximately $\frac{\sin C\, \delta A}{\sin A \, . \, \sin(A + C)}$.

(8 marks)

(ii) Investigate the maximum and minimum values of the expression :

$x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$.

(7 marks)

Turn over

III. (a) (i) Discuss the convergence of the series $\sum_{n=1}^{\infty}\left(\frac{2^n - 2}{2^n + 1}\right)x^{n-1}\ (x > 0)$. (8 marks)

(ii) Determine the interval of convergence for the series $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots$ (7 marks)

Or

(b) (i) Test the convergence of the series $\sum_{n=1}^{\infty}\frac{x^{2n-2}}{(n+1)\sqrt{n}}$ for $x > 0$. (8 marks)

(ii) Discuss the convergence of the series $\sum_{n=1}^{\infty}\frac{(n+1)^n x^n}{n^{n+1}}$. (7 marks)

IV. (a) (i) Using Cayley-Hamilton theorem find A^{-1} for $A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$. (8 marks)

(ii) Find the nature of the quadratic form $3x^2 - 2y^2 - 2z^2 - 4xy + 8xz + 12yz$. (7 marks)

Or

(b) Reduce $8x^2 + 7y^2 + 3z^2 - 12xy - 8yz + 4xz$ into a canonical form by an orthogonal reduction. Also find the nature of the quadratic form. (15 marks)

V. (a) (i) Expand $f(x) = x - x^2, -\pi \le x \le \pi$ in Fourier series. Hence show that :

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots = \frac{\pi^2}{12}$$

(8 marks)

(ii) Expand $f(x) = x(\pi - x),\ 0 \le x \le \pi$ in a Fourier cosine series. (7 marks)

Or

(b) (i) Obtain the constant term and the coefficients for the first sine and cosine terms in the Fourier series that represents y as given in the following table :—

x	:	0	1.0	2.0	3.0	4.0	5.0	6.0
y	:	9	18	24	28	26	20	9

(8 marks)

(ii) Express $f(x) = x \sin x,\ 0 < x < \pi$ in a half range sine series. (7 marks)

[4 × 15 = 60 marks]