D 10025 (Pages : 2) Name.....................................

Reg. No.................................

THIRD SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION NOVEMBER 2010

EN 09 301—ENGINEERING MATHEMATICS—III

Time : Three Hours Maximum : 70 Marks

Part A

Answer all *questions.*

1. Prove that $f(z) = z\,I_{mz}$ is differentiable only at $z = 0$.

2. Obtain the expansion of $\frac{z-1}{z^2}$ in a Taylor series in powers of $(z-1)$.

3. Find the invariant points of the transformation $w = \frac{3z-5i}{iz-1}$.

4. Determine the subspace spanned by $H = \{(1,-2,3),(3,0,1)\}$ in $\mathbb{R}^3$.

5. Find the Fourier sine transform of

$$f(x) = \begin{cases} 1, & 0 < x < a \\ 0, & x \geq a \end{cases}.$$

(5 × 2 = 10 marks)

Part B

Answer any four *questions.*

6. Find the image of the line $y - x + 1 = 0$ under the transformation $w = \frac{1}{z}$.

7. Evaluate $\int_C \frac{3z^2+7z+1}{3+1}\,dz$ where C is the circle $|z+i| = 1$.

8. Let $V = \mathbb{R}^4$ and W be a subspace generated by (1, –2, 5, –3), (2, 3, 1, –4) and (3, 8, –3, –5). Find a basis and dim (W).

9. Show that the set $\{(x, y, z) \in \mathbb{R}^3 : 2x + 5y - 7z = 0\}$ is a vector subspace of $\mathbb{R}^3$.

10. Find the Fourier transform of the function $f(t) = e^{-a|t|}, -\infty < t < \infty, a > 0$.

11. Find the Fourier cosine transform of $f(x) = \frac{e^{-ax}}{x}$.

(4 × 5 = 20 marks)

Turn over

Part C

Answer **all** *questions as per choice given.*

12. (a) Find the bilinear transformation which maps the points $z = 1, i, -1$ on to the points $w = i, 0, -i$. Hence find the image of $|z| < 1$.

Or

(b) Determine the analytic function whose real part is $\frac{\sin 2x}{\cosh 2y - \cos 2x}$.

13. (a) Evaluate $\oint_C \frac{e^z dz}{\cos \pi z}$ where C is the unit circle $|z| = 1$.

Or

(b) Evaluate $\int_0^{2\pi} \frac{d\theta}{5 - 3\cos\theta}$

14. (a) Let $u_1 = (1, 0, 0)$ $u_2 = (-3, 4, 0)$ and $u_3 = (3, -6, 3)$.

(i) Show that $B = \{u_1, u_2, u_3\}$ is a basis of $\mathbb{R}^3$.

(ii) Find the co-ordinate vector for $v = (-8, 2, 3)$.

Or

(b) Find an orthonormal basis for the sub-space spanned by (1, 2, 1), (1, 0, 1), (3, 1, 0) of $\mathbb{R}^3$.

15. (a) Using the Fourier Integral representation, show that $\int_0^\infty \frac{w \sin xw}{1 + w^2} dw = \frac{\pi}{2} e^{-x}, (x > 0)$.

Or

(b) Show that the Fourier transform of $e^{-x^2/2}$ is self reciprocal.

(4 × 10 = 40 marks)