C 26471 (Pages : 3) Name.....................................

Reg. No...................................

COMBINED FIRST AND SECOND SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, APRIL 2012

PTEN/EN 09 102—ENGINEERING MATHEMATICS —II

Time : Three Hours Maximum : 70 Marks

Part A

Answer **all** *questions.*

1. Solve $\left(x^2 - ay\right)dx = \left(ax - y^2\right)dy$

2. Find the Laplace transform of

$$f(t) = k, \ 0 \le t \le a$$
$$= -k, a \le t \le 2a$$

and $f(t + 2a) = f(t) \ \forall \ t.$

3. Find grad ϕ if $\phi = \left(y^2 - 2xyz^3\right)\hat{i} + (3 + 2xy - x^2 z^3)\hat{j} + (6z^3 - 3x^2 y z^2)\hat{k}$.

4. If $\vec{k} = \left(x^2 + y^2 + 2xz\right)\overline{i} + \left(xz - xy + yz\right)\overline{j} + \left(z^2 + x^2\right)\overline{k}$, find $\nabla \cdot \vec{F}$.

5. Evaluate $\int_C \phi \, d\,\overline{r}$, where C is the curve $x = t, y = t^2, z = (1 - t)$ and $\phi = x^2 \, y\,(1 + z)$ from $t = 0$ to $t = 1$.

(5 × 2 = 10 marks)

Part B

Answer any **four** *questions.*

6. Solve $\frac{y}{x}\frac{dy}{dx} + \frac{x^2 + y^2 - 1}{2(x^2 + y^2) + 1} = 0$.

7. Solve $L\left\{\sin h\,\frac{t}{2}\sin\frac{\sqrt{3}}{2}t\right\}$.

Turn over

9. Find the angle between the surfaces $x^2 - y^2 - z^2 = 11$ and $xy + yz - zx = 18$ at the point (6, 4, 3).

10. Evaluate $\int_C \left(x^2 - y^2\right) dx + 2xydy$ where C is the boundary of the rectangle in the xoy -plane bounded by the lines $x = 0$, $x = a$, $y = 0$ and $y = b$.

11. Use Gauss divergence theorem for $\overline{F} = x^2\,\overline{i} + y^2\,\overline{j} + z^2\,\overline{k}$, where S is the surface of the cuboid formed by the planes $x = 0$, $x = a$, $y = 0$, $y = b$, $z = 0$ and $z = c$ for evaluating $\iint_S \overline{F} \cdot d\,\overline{S}$.

(4 × 5 = 20 marks)

Part C

Answer section (a) **or** *section (b) of each question.*

12. (a) Solve $(D^2 - 4D + 4)\,y = 8x^2\,e^{2x}\,\sin x$

Or

(b) Solve the equation $\left(x^2 + 1\right)\frac{dy}{dx} + 4xy = \frac{1}{x^2 + 1}$ by using method of variation of parameter.

13. (a) Find the Laplace transform of $\int_0^\infty \left(\frac{\cos at - \cos bt}{t}\right) dt$.

Or

(b) Find the inverse Laplace transform of $\frac{s^2 + 8s + 16}{\left(s^2 + 6s + 10\right)^2}$.

14. (a) Show that $\overline{F} = (y^2 - z^2 + 3yz - 2x)\hat{i} + (3xz + 2xy)\bar{j} + (3xy - 2xz + 2z)\bar{k}$ is both solenoidal and irrotational.

Or

(b) If u and v are scalar point functions and $\overline{F}$ is a vector point function such that $u\,\overline{F} = \nabla v$, prove that $\overline{F} \cdot \text{curl}\, \overline{F} = 0$.

15. (a) If S is a closed surface enclosing a volume V, evaluate $\iint_S \nabla (r^2) \cdot d\,\overline{S}$

Or

(b) Use Green's theorem in a plane to find the area of the region in the xoy -plane bounded by $y^3 = x^2$ and $y = x$.

(4 × 10 = 40 marks)