C 6284 (Pages : 2) Name..................................

Reg. No..............................

COMBINED FIRST AND SECOND SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, MAY 2010

PTEN/EN 09 102—ENGINEERING. MATHEMATICS—II

(2009 admissions)

Time : Three Hours Maximum : 70 Marks

Part A

Answer **all** *questions.*

1. Solve $\left(x^2 - ay\right)dx = \left(ax - y^2\right)dy.$

2. Solve $\left(D^2 + 2D - 1\right)y = \left(x + e^x\right)^2.$

3. Find $L\left(\cos^2 t\right).$

4. Show $\nabla \times \bar{F} = 0$ if $\bar{F} = \left(y^2 + 2xz^2\right)\bar{i} + (2xy - z)\bar{j} + \left(2x^2z - y + 2z\right)\bar{k}.$

5. State Gauss Divergence theorem.

(5 × 2 = 10 marks)

Part B

Answer any **four** *questions.*

6. Solve $\left(D^2 + 4\right)y = x^4 + \cos^2 x.$

7. Solve $\left(1 + e^{x/y}\right)dy + e^{x/y}\left(1 - \frac{x}{y}\right)dy = 0.$

8. Find the Laplace transform of $\left(t^3 + 3e^{2t} - 5\sin 3t\right)e^{-t}.$

9. Find $L^{-1}\left\{\frac{1}{s^2\left(s^2 + a^2\right)}\right\}.$

10. If $\vec{a}$ is a constant vector and $\vec{r}$ is the position vector of the point (x, y, z) with respect to the origin, prove that grad $\left(\vec{a}.\vec{r}\right) = \vec{a}.$

Turn over

11. Evaluate $\oint\limits_{C} \bar{F}\cdot d\bar{r}$ where $\bar{F}(\sin y)\hat{i} + (1+\cos y)\,j + z\bar{k}$ and C is the circle $x^2 + y^2 = a^2$ in the xy-plane.

(4 × 5 = 20 marks)

Part C

Answer section (a) **or** *section (b) of each question.*

Each question carries 10 *marks.*

12. (a) Solve $\left(D^4 - 2D^3 + D^2\right) y = x^2 + e^x$.

Or

(b) Solve the equation $\frac{d^2y}{dx^2} + a^2 y = \tan ax$, by the method of variation of parameters.

13. (a) Given that $L^{-1}\left\{\frac{1}{\left(s^2+4\right)}\right\} = \frac{1}{16}(\sin 2t - 2t\cos 2t)$, find $L^{-1}\left\{\frac{1}{\left(s^2+a\right)^2}\right\}$.

Or

(b) Find the Laplace transform of $L\{t \sin at\}^2$.

14. (a) If $u = x^2yz$ and $V = xy - 3z^2$, find $\nabla \times (\nabla u \times \nabla v)$ at the point (1, 1, 0).

Or

(b) Prove that $\bar{F} = 3y2\bar{i} + 2zx\bar{j} + 4xy\bar{k}$ is not irrotational, but $\left(x^2yz^3\right)\bar{F}$ is irrotational.

15. (a) Evaluate $\oint\limits_{C} (yzdx + zxdy + xydz)$ where C is the circle given by $x^2 + y^2 + z^2 = 1$.

Or

(b) Use divergence theorem to evaluate $\iint\limits_{s} \left(4x\hat{i} + \left(-2y^2\right)\hat{j} + z^2\hat{k}\right).d\bar{S}$, where S is the closed surface bounded by the cylinder $x^2 + y^2 = 4$ and the plane $z = 0$ and $z = 3$.

(4 × 10 = 40 marks)